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Bulk fermions in multi-brane worlds

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Abstract: We study bulk fermion fields in various multi-brane models with localized gravity. The chiral zero mode that these models support can be identified as a right-handed sterile neutrino. In this case small neutrino Dirac masses can naturally appear due to a localization of the bulk fermion zero mode wavefunction, in an analogous way to graviton, without invoking the see-saw mechanism. The conditions and the options for localization are discussed in detail. It is shown that, considering a well motivated five dimensional mass term, the localization behaviour of this mode can resemble the graviton's at least in a region of the parameter space. As a result, the "$+-+$", "$++$" models can support, in addition to the ultralight graviton KK state, an ultralight localized and strongly coupled bulk fermion KK mode. We find that there are severe constraints on the parameter space of "$+-+$" and "$++$" models if the neutrino properties resulting from this light fermion state are to be reasonable. Furthermore, in the case that also the Bigravity scenario is realized the above special KK mode can induce too large mixing between the neutrino and the KK tower sterile modes restricting even more the allowed parameter space.

Keywords: Field Theories in Higher Dimensions, Neutrino Physics.

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1. Introduction

The study of bulk fermion fields, although not something new [1, 2], turns out to be of particular interest in the context of brane-world[1] scenarios both in the case of models with large extra dimensions [4]–[6] (factorizable geometry) and in models of localized gravity [7]–[10] (non factorizable geometry) since they can provide possible new ways to explain the smallness of the neutrino masses, neutrino oscillations and the pattern of fermion mass hierarchy.

In the context of string and M-theory, bulk fermions arise as superpartners of gravitational moduli, such as, those setting the radii of internal spaces. Given this origin, the existence of bulk fermions is unavoidable in any supersymmetric string compactification and represents a quite generic feature of string theory.[2] This constitutes the most likely origin of such particles within a fundamental theory and, at the same time, provides the basis to study brane-world neutrino physics.

[1]The idea dates back to early eighties [3].

[2]However, note that brane-world models with non factorizable geometry have not yet been shown to have string realizations. For string realizations of models with large extra dimensions see ref. [6].

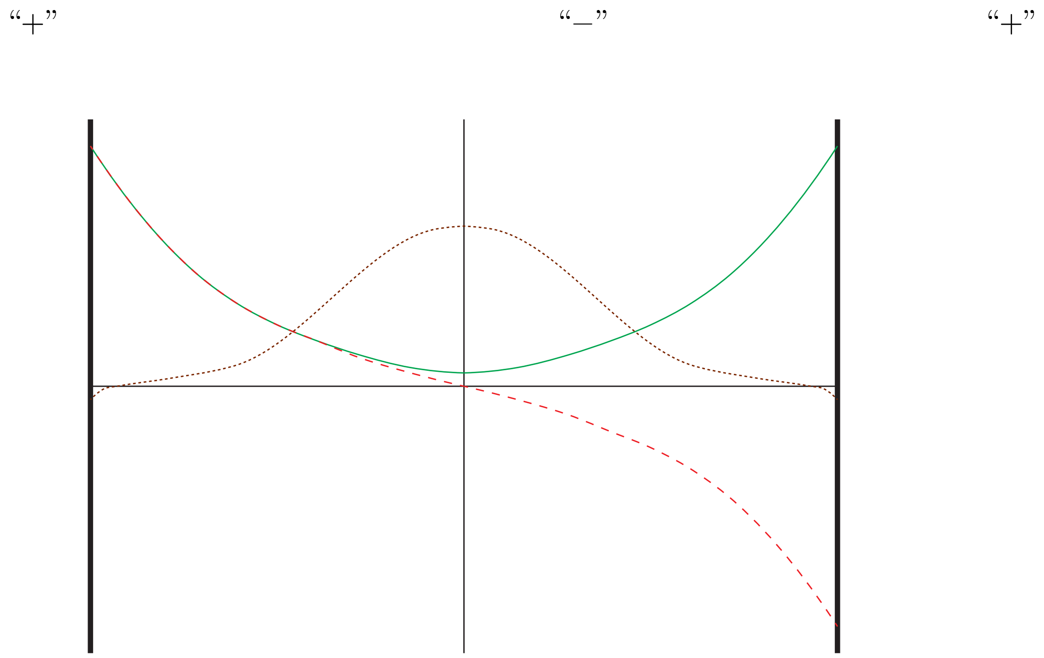

Figure 1: The right-handed fermion zero mode (solid line), first (dashed line) and second (dotted line) KK states wavefunctions in the symmetric "$+-+$" model. The same pattern can occur for the corresponding wavefunctions in the "$++$" model. The wavefunctions of the zero and the first KK mode are localized on the positive tension branes. Their absolute value differ only in the central region where they are both suppressed resulting to a very light first KK state.

In the traditional approach the small neutrino masses are a result of the see-saw mechanism, in which a large right-handed Majorana mass M_R suppresses the eigenvalues of the neutrino mass matrix leading to the light neutrino mass $m_\nu \sim m^2_{\text{fermion}}/M_R$. The neutrino mixing explanations of the atmospheric and solar neutrino anomalies require that M_R to be a superheavy mass scale $> 10^{10}$ GeV.

In the case of large extra dimensions [4]–[6], despite the absence of a high scale like M_R (since in such models the fundamental scale can be as low as 1 TeV), small neutrino masses [11, 12] (Dirac or Majorana) can arise from an intrinsically higher-dimensional mechanism. The idea is that any fermionic state that propagates in the bulk, being a Standard Model (SM) singlet can be identified with a sterile neutrino which through it's coupling to the SM left-handed neutrino can generate small neutrino mass. In the case of factorizable geometry, the smallness of the induced masses is due to the fact that the coupling is suppressed by the large volume of the internal bulk manifold. In other words, the interaction probability between the bulk fermion zero mode, the Higgs and Lepton doublet fields (which are confined to a brane) is small because of the large volume of bulk compared to the thin wall where the SM states are confined, resulting a highly suppressed coupling. In the context of these models one can attempt to explain the atmospheric and solar neutrino anomalies (see e.g. [11], [13]–[17]).

In the context of brane world models with localized gravity [7]–[9] (non factorizable geometry) small neutrino masses can again be achieved, without invoking the see-saw mechanism. In more detail, in this case the mechanism generating the small coupling between the Lepton doublet and the Higgs which live on the brane and the right-handed sterile neutrino zero mode is not due to the compactification volume (which is now small) but due to the fact that the sterile neutrino wavefunction can be localized [18-30] on a distant brane. One may thus arrange that the overlap between this mode and the SM brane is sufficiently small. In this case the AdS_5 geometry localizes the fermion zero mode on negative tension branes. Localization can occur on positive branes if a mass term of the appropriate form is added to counterbalance the effect of the AdS_5 geometry, by applying the ideas presented in ref. [1, 2]. Such a mass term appears naturally in the case that, the branes arise as the limiting cases of domain walls that are created from a five-dimensional scalar field with an nontrivial ground state (kink or multi-kink) [25, 26]. In this case the scalar field naturally couples to the bulk fermion through an non trivial "mass" term and will naturally induce localization to the bulk modes on positive or negative branes - which depends on the vacuum expectation value of the scalar field.

In this paper we analyze the localization of a bulk fermion the mass spectrum and the coupling between SM neutrino and bulk states in the context of multi-brane worlds (e.g. see ref. [10]). We discuss in detail the conditions and the options for the localization in relation to the form of the bulk mass term. We also discuss the possibility of generating small neutrino masses in the context of "$++-$", "$+-+$",[3] "$++$" models. We determine for which regions of the parameter space lead to a solution of the hierarchy problem and generation of small neutrino masses. The study of "$+-+$" and "$++$" models reveals the possibility of an ultralight KK state of the bulk fermion analogous to the KK graviton in the gravitational sector [31]–[34]. This is due to the fact that, in this region the wavefunction of the right-handed bulk fermion states obeys a similar equation to that of the graviton. The above fermion state, when exists, imposes even more severe constraints on the parameter space of these models.

The organization of the paper is as follows: In the next section 2 we review the general formalism, and discuss the form of the bulk neutrino mass term which turns out to be critical in determining the neutrino properties. In section 3 we review the bulk fermion properties in the Randall-Sundrum (RS) model [8]. In section 4 we extend this to the case of three brane "$++-$" model [33]. In sections 5 and 6 we study in detail the "Bigravity" three brane models "$+-+$" [31, 32] and "$++$" [34]. In section 7 we discuss the implications of the realization of the "Bigravity" in relation to the presence of bulk spinors. The overall implications and conclusions are presented in section 8.

[3] We consider the "$+-+$" configuration as a toy-model ignoring the phenomenological difficulties associated with the presence of a negative tension brane [36] since it's phenomenology is very similar to the "$++$" which includes only positive tension branes.

2. General framework

Following the framework introduced in ref. [19], we consider a spinor Ψ in a five dimensional AdS_5 space-time, where the extra dimension is compact and has the geometry of an orbifold $S^1/\mathbb{Z}_2$. The AdS_5 background geometry is described by:[4]

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \tag{2.1}$$

where the warp factor $\sigma(y)$ depends on the details of the model considered. For the moment we assume that we have a model with a number of positive and negative tension flat branes (the sum of the brane tensions should be zero if one wants flat four dimensional space on the branes) and that this function is known (it can be found by looking the system gravitationally).

The action for a Dirac Spinor of a mass m in such a background is given by:[5]

$$S = \int d^4x \int dy\sqrt{G}\left\{E^A_\alpha\left[\frac{i}{2}\bar{\Psi}\gamma^\alpha\left(\overrightarrow{\partial_A} - \overleftarrow{\partial_A}\right)\Psi + \frac{\omega_{bcA}}{8}\bar{\Psi}\{\gamma^\alpha, \sigma^{bc}\}\Psi\right] - m\Phi\bar{\Psi}\Psi\right\}, \tag{2.2}$$

where $G = \det(G_{AB}) = e^{-8\sigma(y)}$. The four dimensional representation of the Dirac matrices in five-dimensional flat space is chosen to be: $\gamma^\alpha = (\gamma^\mu, i\gamma_5)$. The inverse vielbein is given by $E^A_\alpha = \mathrm{diag}(e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, 1)$. Due to the fact that the vielbein is symmetric, the contraction of ω_{bcA}.[6] with the corresponding term in the action gives vanishing contribution. The mass term is assumed to be generated by a Yukawa coupling with a scalar field Φ which has a nontrivial stable vacuum $\Phi(y)$.

2.1 The mass term

A few comments on presence of the mass term are in order. The motivation for introducing a mass term of this form comes from the need to localize fermion zero modes in the extra dimension. This is discussed in more details in the next paragraph. Note that the "localization" of the wavefunction of a state does not necessarily reflect the actual localization of the state, since in one has to take in account the nontrivial geometry of the extra dimension - something that is done when we calculate physical quantities. Also when we note that a state is localized on a brane we mean that this holds irrespectively if the space is compact or not (thus the state should be normalizable even in the non-compact case). In order to have a localized state (zero mode) on a positive brane it is necessary to have an appropriate bulk mass term. This is because [18, 20] the background AdS_5 geometry itself has the opposite effect

[4] We will assume that the background metric is not modified by the presence of the bulk fermion, that is, we will neglect the back-reaction on the metric from the bulk fields.

[5] We do not include a Majorana mass term, $\Psi^T C\Psi$, which is forbidden if the bulk fermion has a conserved lepton number.

[6] $\omega_{\mu ab} = 1/2(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu})e_a{}^\nu - 1/2(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})e_b{}^\nu - 1/2 e_a{}^\rho e_b{}^\sigma(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e^c{}_\mu$, where $e_\mu{}^\nu$ is the vielbein.

favouring localization on negative tension branes. As we will discuss, this leads to a critical mass m_{cr} below which the localization is still on the negative brane (if we consider a configuration of a positive and one negative brane), for $m = m_{cr}$ the is no localization and for $m > m_{cr}$ the zero mode is localized on the positive tension brane.[7]

Let us return to the specific form of the mass term and it's generality since this will become important in later discussions. Since the mass term is a key element for the localization let us, for a moment assume that it has the form $m\Phi(y)\bar{\Psi}\Psi$ where $\Phi(y)$ is the vev of a scalar field which has a nontrivial stable vacuum (its vev does not depend on the remaining spatial dimensions). Now if we do a simple calculation, in the case of flat extra dimension (the general arguments will apply also in our background eq. (2.1), taking in account also the effects of the AdS_5 geometry), we find that the above configuration implies that the zero mode satisfies a Schröndiger equation with potential of the form $V(y) = \Phi^2(y) - \Phi'(y)$. In order to localize one state the profile of $\Phi(y)$ should be such that it creates a potential well. The way to do this is to assume that the ground state of the scalar field has a kink or a multi-kink profile [1, 2, 25]. Although the details of these profiles depend on the form of the potential of the scalar field Φ, if we demand strong localization of the states, the kink profiles tend to θ- functions. This implies that the function Φ in eq.(2) can be considered as an arbitrary combination of θ-functions (compatible with the symmetries of the action). However, as shown in ref. [25] the same field Φ can be used in order to create the branes themselves. This restricts the possible form of the mass term. If we assume that the same field Φ creates the branes and localizes the fermion zero mode the mass term should have a (multi-)kink form, with $\Phi(y) = \sigma'(y)/k$ up to a sign.[8] The previous argument also supports the θ-function form of the mass term (and not for example a $\tanh(y)$ profile) since we assume that the branes are infinitely thin. Note thought that in the "++" model due to the AdS_4 geometry on the branes the, $\sigma'(y)$ function does not have just a θ-function form but it also involves kink profiles.

If one assumes that the mass is generated by coupling to a scalar field, different from the one that creates the branes, it can have any form allowed by the

[7]One may ask why one should localize the fermion zero mode on a positive tension brane and not on a negative ? In the case of RS model it is obvious that if we demand to solve the hierarchy problem and in the same time to create small neutrino masses through this mechanism, one should localize the fermion zero mode on the positive brane. In the case of multi-brane models this is not a necessity and thus, in principle, another possibility (of course in this case the mass term - if needed - should have different form (e.g. for $m \to -m$)).

[8]Note thought, that if we choose the opposite sign for the mass term (i.e. $m \to -m$) the localization of the fermion zero mode will always be on the negative tension branes. Taking in account the fact that the latter occurs also, in a region of the parameter space, in the case that our mass term choice is the one that appears in eq.(2), we will not consider this possibility separately since we can easily, as we will see, generalize our results for $m < 0$ (the presence of such a mass term sharpens the localization of the states on negative branes).

dynamics and the symmetries of the action. For example it can take the form: $-m\,(\theta(y)-\theta(-y)))$. In this case, it will tend to induce localization on the brane sitting at the origin of the orbifold. Nevertheless this will not be satisfactory option in multi-brane models where the desired M_{Pl}/M_{EW} hierarchy is not generated between the two first branes (e.g. "$+-+$" or even in the "++" model) since in these cases it will not generally be possible to simultaneously solve the hierarchy problem and generate small neutrino masses (it would work though in the "$++-$" model - or it could work in the cases where the background induced localization dominates but in that case the mass term would be unnecessary anyway). In any case since we are interested in the most economic, in terms of parameters and fields, models we will not consider these possibilities.

The geometry we consider has a $\mathbb{Z}_2$ symmetry ($y \to -y$). Under this the fermion parity is defined as: $\Psi(-y) = \gamma_5\Psi(y)$ (i.e. $\Psi_L(-y) = -\Psi_L(y)$, $\Psi_R(-y) = \Psi_R(y)$) and changes the sign of a lagrangian mass term of the form: $m\bar{\Psi}\Psi = m(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L)$. The full mass term however is invariant under the $\mathbb{Z}_2$ since the function $\sigma'(y)$ is also odd under the reflections $y \to -y$. With this definition of parity one of the wavefunctions will be symmetric and the other antisymmetric with respect to the center of the orbiford . Note that this implies that the odd wavefunction will be zero at the orbifold fixed points (i.e. zero coupling to fields confined to that points). Since we would like in what follows to use the right-handed component in order to give mass to SM neutrinos, which could be confined on a brane at an orbifold fixed point, we choose the right-handed wavefunction to be even (i.e. non-vanishing coupling) and the left-handed to be odd.

2.2 The KK decomposition

It is convenient to write the action in terms of the fields: Ψ_R and Ψ_L where $\Psi_{R,L} = 1/2(1\pm\gamma_5)\Psi$ and $\Psi = \Psi_R + \Psi_L$. The action becomes:

$$S = \int d^4x \int dy \Big\{ e^{-3\sigma}\left(\bar{\Psi}_L i\gamma^\mu\partial_\mu\Psi_L + \bar{\Psi}_R i\gamma^\mu\partial_\mu\Psi_R\right) - e^{-4\sigma} m\left(\frac{\sigma'(y)}{k}\right)\times \tag{2.3}$$
$$\times\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L\right) - \frac{1}{2}\left[\bar{\Psi}_L(e^{-4\sigma}\partial_y + \partial_y e^{-4\sigma})\Psi_R - \bar{\Psi}_R(e^{-4\sigma}\partial_y + \partial_y e^{-4\sigma})\Psi_L\right]\Big\}$$

writing Ψ_R and Ψ_L in the form:

$$\Psi_{R,L}(x,y) = \sum_n \psi_n^{R,L}(x) e^{2\sigma(y)} f_n^{R,L}(y) \tag{2.4}$$

the action can be brought in the form

$$S = \sum_n \int d^4x \{\bar{\psi}_n(x) i\gamma^\mu\partial_\mu\psi_n(x) - m_n\bar{\psi}_n(x)\psi_n(x)\} \tag{2.5}$$

provided the wavefunctions obey the following equations

$$\begin{aligned}\left(-\partial_y + m\frac{\sigma'(y)}{k}\right) f_n^L(y) &= m_n e^{\sigma(y)} f_n^R(y) \\ \left(\partial_y + m\frac{\sigma'(y)}{k}\right) f_n^R(y) &= m_n e^{\sigma(y)} f_n^L(y)\end{aligned} \tag{2.6}$$

and the orthogonality relations (taking account of the $\mathbb{Z}_2$ symmetry):

$$\int_{-L}^{L} dy e^{\sigma(y)} {f^{L}}^*_m(y) f_n^L(y) = \int_{-L}^{L} dy e^{\sigma(y)} {f^{R}}^*_m(y) f_n^R(y) = \delta_{mn}\,, \tag{2.7}$$

where we assume that the length of the orbifold is $2L$.

We solve the above system of coupled differential equations by substituting $f_n^L(y)$ from the second in the first equation. Thus we end up with a second order differential equation, which can always be brought to a Schrödinger form by a convenient coordinate transformation from y to z coordinates related through $dz/dy = e^{\sigma(y)}$, the coordinate transformation chosen to eliminate the terms involving first derivatives. Thus we end up with the differential equation of the form:

$$\left\{-\frac{1}{2}\partial_z^2 + V_R(z)\right\} \hat{f}_n^R(z) = \frac{m_n^2}{2}\hat{f}_n^R(z) \tag{2.8}$$

$$\text{with} \qquad V_R(z) = \frac{\nu(\nu+1)(\sigma'(y))^2}{2[g(z)]^2} - \frac{\nu}{2[g(z)]^2}\sigma''(y)\,. \tag{2.9}$$

Here $\hat{f}_n^R(z) = f_n^R(y)$ and we have defined $\nu \equiv m/k$ and $g(z) \equiv e^{\sigma(y)}$. The left handed wavefunctions are given by:[9]

$$f_n^L(y) = \frac{e^{-\sigma(y)}}{m_n}\left(\partial_y + m\frac{\sigma'(y)}{k}\right) f_n^R(y)\,. \tag{2.10}$$

The form of eq. (2.8) and (2.9) are exactly the same as that satisfied by the graviton when $\nu = 3/2$ [31]. For any ν, we note that before orbifolding the system supports two zero modes (left-handed and right-handed) [19]. However, the orbifold compactification leaves only a chiral right-handed zero mode.

We note that since the bulk fermion mass, m, is a parameter that appears in the original five dimensional lagrangian its "natural" value is of the order of the five dimensional Planck scale M_5. Now since we assume that $k < M_5$ (in order to trust our perturbative analysis when we consider the configuration gravitationally) it is clear that the "physical" value of ν is $\nu > 1$. However, we will always comment on the behaviour of our results out of this region (even for negative values).

[9]Note that it can be shown that the left-handed component obeys also a similar Schrödinger equation with $V_L(z) = \frac{\nu(\nu-1)k^2}{2[g(z)]^2} + \frac{\nu}{2[g(z)]^2}\sigma''(y)$ which is the same as V_R for $\nu \to -\nu$.

We are particularly interested in the coupling of the bulk spinor to the SM neutrinos since this is the way that the neutrino masses will be generated. In order to avoid weak scale neutrino masses and lepton number violating interactions we assign lepton number $L = 1$ to the bulk fermion state and thus the only gauge invariant coupling is of the form

$$S_Y = -\int d^4x\sqrt{-g_{Br}}\{Y_5\bar{L}_0(x)\widetilde{H_0}(x)\Psi_R(x, L_{Br}) + \text{h.c.}\}\,, \tag{2.11}$$

where H_0 is the SM Higgs field, L_0 is the SM lepton doublet, $\widetilde{H_0} = i\sigma_2 H_0^*$, $g_{\mu\nu}^{Br}$ is the induced metric on the brane and $g_{Br} = \det(g_{\mu\nu}^{Br})$. The Yukawa parameter Y_5 has mass dimension $-1/2$ and thus since it appears a parameter in the five dimensional action, its "natural" value is $Y_5 \sim 1/\sqrt{M_5} \sim 1/\sqrt{k}$.

To obtain canonical normalization for the kinetic terms of the SM neutrino we perform the following field rescalings $H_0 \to e^{\sigma(L_{Br})}H$, $L_0 \to e^{3\sigma(L_{Br})/2}L$, where L_{Br} is the position of the brane that SM is confined. This gives

$$S_Y = -\sum_{n\geq 0}\int d^4x\{y_n\bar{L}(x)\widetilde{H}(x)\psi_n^R(x) + \text{h.c.}\}\,, \tag{2.12}$$

where

$$y_n \equiv e^{\sigma(L_{Br})/2}\ Y_5\ f_n^R(L) = (g(z_{Br}))^{1/2}\ Y_5\ \hat{f}_n^R(z_L)\,. \tag{2.13}$$

From the above interaction terms we can read off the mass matrix $\mathbf{M}$ that appears in the lagrangian as $\bar{\psi}_L^\nu\mathbf{M}\psi_R^\nu + \text{h.c.}$ where we have defined $\psi_L^\nu = (\nu_L, \psi_1^L, \ldots, \psi_n^L)$ and $\psi_R^\nu = ({\psi_0}^R, \psi_1^R, \ldots, \psi_n^R)$. The mass matrix for the above class of models has the following form

$$\mathbf{M} = \begin{pmatrix} vy_0 & vy_1 & \ldots & vy_n \\ 0 & m_1 & \ldots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \ldots & m_n \end{pmatrix}. \tag{2.14}$$

3. Neutrinos in RS model

For completeness of our analysis, we first briefly review the case of bulk fermion spinors in the RS model [19]. This model consists of one positive (hidden) and one negative tension brane (where the SM fields are confined) placed on the fixed points ($y = 0,\ L_1$) of a $S^1/\mathbb{Z}_2$ orbifold (for details see [8]). In this case the background geometry is described by eq. (2.1) where $\sigma(y) = k|y|$. The convenient choice of variable, for the reasons described in the previous section, is:

$$z \equiv \frac{e^{ky} - 1}{k} \qquad y \in [0, L_1]\,. \tag{3.1}$$

Since in this model we have $(\sigma'(y))^2 = k^2$ and $\sigma''(y) = 2kg(z)\left[\delta(z) - \delta(z - z_1)\right]$, the potential appearing in eq. (2.8) of the Schrödinger equation that the wavefunction of the right-handed bulk fermion is (for $z \geq 0$):

$$V_R(z) = \frac{\nu(\nu+1)k^2}{2[g(z)]^2} - \frac{\nu}{2g(z)} 2k\left[\delta(z) - \delta(z - z_1)\right]. \tag{3.2}$$

Here we have defined $g(z) \equiv kz + 1$ and $z_1 \equiv z(L_1)$.

This potential always gives rise to a (massless) zero mode. It is given by

$$\hat{f}_0^R(z) = \frac{A}{[g(z)]^\nu}. \tag{3.3}$$

From the above expression it seems that the zero mode is always localized on the positive tension brane for all values of ν. Nevertheless, by taking the second brane to infinity, we find that the zero mode is normalizable in the case that $\nu > 1/2$ and that it fails to be normalizable when $0 \leq \nu \leq 1/2$. The above, as we mentioned, shows that only when $\nu > 1/2$ the zero mode is localized on the first brane. For $\nu = 1/2$ there is no localization and for $0 \leq \nu < 1/2$ it is localized on the negative brane. Another way to see the above is to find the coupling of the KK states to mater of a "test" brane as a function of the distance from the first (hidden) brane. From eq.(13) we can find that in the case of $\nu > 1/2$ the coupling decreases as we go away from the first brane, on the other hand it is constant when $\nu = 1/2$ (no localization), and increases when $0 \leq \nu < 1/2$ (localization on the second brane). In any case as we previously mentioned the "natural" value for ν can be considered to be greater than unity (having already restricted ourselves in the region $\nu > 0$) and thus we will assume in the following discussions that the right-handed zero mode is always localized on the hidden positive tension brane and we will briefly discuss the rest possibilities. Note that all the following expressions for the masses and the coupling are valid under the assumption that $\nu > 1/2$, as the results for the rest of the parameter space are different. In this case the normalization constant is $A \simeq \sqrt{k(\nu - 1/2)}$.

Apart from the zero mode we have to consider the left and right-handed KK modes which correspond to solutions for $m_n > 0$. The solutions for the right-handed wavefunctions in this case are given in terms of Bessel functions:[10]

$$\hat{f}_n^R(z) = \sqrt{\frac{g(z)}{k}}\left[AJ_{\nu+1/2}\left(\frac{m_n}{k}g(z)\right) + BJ_{-\nu-1/2}\left(\frac{m_n}{k}g(z)\right)\right]. \tag{3.4}$$

[10]Note that in the case that $\nu = N + 1/2$, where N is an integer, the two linearly independent solutions are: J_{N+1} and Y_{N+1}. Although for our calculations we have assumed that $\nu \neq N + 1/2$, all the results for the mass spectrum and the couplings are valid also in the special cases when $\nu = N + 1/2$.

These solutions must obey the following boundary conditions:

$$\begin{aligned} \hat{f}_n^{R\,\prime}(0^+) + \frac{k\nu}{g(0)} \hat{f}_n^R(0) &= 0 \\ \hat{f}_n^{R\,\prime}({z_1}^-) + \frac{k\nu}{g(z_1)} \hat{f}_n^R(z_1) &= 0\,. \end{aligned} \tag{3.5}$$

The wave functions of the left-handed KK states can be easily extracted from eq. (2.10). The boundary conditions give a 2 x 2 system for A,B which, in order to have a nontrivial solution, should have vanishing determinant. This gives the quantization of the spectrum. For $\nu > 1/2$ the quantization condition can be approximated by a simpler one: $J_{\nu-1/2}\left(m_n g(z_1)/k\right) = 0$ This implies that the KK spectrum of the bulk state is:

$$m_n = \xi_n \; k \; e^{-kL_1} \tag{3.6}$$

(for $n \geq 1$), where ξ_n in the n-th root of $J_{\nu-1/2}(x)$. This means that if one is interested in solving the hierarchy in the context of this model, i.e. $w \equiv e^{-kL_1} \sim 10^{-15}$ the mass of the first bulk spinor KK state will be of the order of 1 TeV and the spacing between the tower will be of the same order. To, summarize the spectrum in this case consists of a chiral right-handed massless zero mode and a tower of Dirac KK states with masses that start from 1 TeV (if a solution of the hierarchy is required) with $\sim$ 1 TeV spacing. The other important point for the phenomenology is the coupling of the bulk spinors to the SM neutrino. It is easy, using eq. (2.13), to find that the zero mode couples as

$$\upsilon\, y_0 = \upsilon\, Y_5 \sqrt{k\left(\nu - \frac{1}{2}\right)}\left(\frac{1}{g(z_1)}\right)^{\nu-1/2} \simeq \upsilon\sqrt{\nu - \frac{1}{2}}\, w^{\nu-1/2} \tag{3.7}$$

since, the hierarchy factor is defined as $w \equiv 1/g(z_1)$ and, as mentioned in the previous section, $Y_5 \sim 1/\sqrt{k}$, $\upsilon \sim 10^2$ GeV and $\nu > 1/2$.

In a similar fashion we can find the couplings of the SM neutrino to bulk KK states. In this particular model it turns out that this coupling does not depend on the fermion mass or the size of the orbifold and thus it is a constant. By a simple calculation we find that

$$\upsilon y_n \;\simeq \sqrt{2}\upsilon\; Y_5\sqrt{k} \simeq \sqrt{2}\;\upsilon\,. \tag{3.8}$$

Thus from the above we see that the KK tower couples to SM neutrino with a TeV strength. In order to find the mass eigenstates and the mixing between the SM neutrino and the sterile bulk modes one has to diagonalize the matrix $\mathbf{MM}^\dagger$ (actually one finds the squares of the mass eigenvalues). By performing the above diagonalization, choosing $e^{-kL_1} \sim 10^{-15}$ it turns out that the mass of the neutrino will be of the order $m_\nu \sim 10^2\;(10^{-15})^{\nu-1/2}$ (e.g. for $\nu = 3/2$, $m_\nu \sim 10^{-4}\,$eV), and the masses of the bulk states are of the order of 1 TeV with a 1 TeV spacing. From the last calculations it appears that one can easily create a small neutrino mass and at the same time arrange for the desired mass hierarchy when $\nu > 1/2$. Apart from creating small masses, one has to check that the mixing between the SM neutrino and the KK tower is small enough so that there is no conflict with phenomenology. It was shown in ref. [19] that this can be done for this model without fine-tuning. Note that the parameter space: $\nu \leq 1/2$ (including negative values) is not of interest in the present discussion[11] since it would be impossible to solve the hierarchy problem and in the same time to assign small masses to neutrinos.

4. Neutrinos in "$++-$" model

Since we are interested in studying the characteristics of bulk fermion modes in multi-brane configurations we add to the "$+-$" RS model another positive tension brane where now SM fields will be confined. Thus we end up with two different configurations: the "$++-$" model which will be the subject of this section and the "$+-+$" model which will be the subject of the next section.

The "$++-$" model consists of two positive and one negative tension brane. The first positive brane is placed on the origin of the orbifold at $y = 0$ the second (where the SM fields are confined), which is freely moving, is place at $y = L_1$ and the negative brane is placed at the second fixed point of the orbifold at $y = L_2$.

In the present model the convenient choice of variables is defined as:

$$z \equiv \begin{cases} \dfrac{2e^{k_1 L_1} - 1}{k_1} & y \in [0, L_1] \\ \dfrac{e^{k_2(y-L_1)+k_1 L1}}{k_2} + \dfrac{e^{k_1 L_1} - 1}{k_1} - \dfrac{e^{k_1 L_1}}{k_2} & y \in [L_1, L_2]\,. \end{cases} \tag{4.1}$$

Note the presence of two bulk curvatures, namely k_1 and k_2 in this model, which is the price that we have to pay in order to place two positive branes next to each other ($k_1 < k_2$ but with $k_1 \sim k_2$ so that we don't introduce another hierarchy. For details see ref. [33]). In terms of the new variables we can find that the potential $V_R(z)$ of the Schrödinger equation that corresponds to the present model has the form (for $z \geq 0$):

$$\begin{aligned} V_R(z) = &\frac{\nu(\nu+1)}{2[g(z)]^2}(k_1^2(\theta(z) - \theta(z - z_1)) + k_2^2(\theta(z - z_1) - \theta(z - z_2))) - \\ &-\frac{\nu}{2g(z)}2\left[k_1\delta(z) + \frac{(k_2 - k_1)}{2}\delta(z - z_1) - k_2\delta(z - z_2)\right] \end{aligned} \tag{4.2}$$

[11] This could be of particular interest if one uses the above mechanism to localize SM fermions on the negative tension brane and in the same time solving the hierarchy problem (e.g. see ref. [23]).

since $\sigma''(y) = 2g(z)\left[k_1\delta(z) + \frac{(k_2-k_1)}{2}\delta(z-z_1) - k_2\delta(z-z_2)\right]$ and $(\sigma'(y))^2 = k_1^2$ for $y \in [0, L_1]$ and $(\sigma'(y))^2 = k_2^2$ for $y \in [L_1, L_2]$. The function $g(z)$ is defined as:

$$g(z) = \begin{cases} k_1 z + 1 & z \in [0, z_1] \\ k_2(z - z_1) + k_1 z_1 + 1 & z \in [z_1, z_2]\,, \end{cases} \tag{4.3}$$

where $z_0 = 0$, $z_1 = z(L_1)$ and $z_2 = z(L_2)$ are the positions of the branes in terms of the new variables.

This potential always gives rise to a (massless) zero mode whose wavefunction is given by

$$\hat{f}_0^R(z) = \frac{A}{[g(z)]^\nu}\,. \tag{4.4}$$

The discussion of the previous section about the state localization applies in this model as well . For $\nu > 1/2$ the zero mode is localized on the first brane. For the case $\nu = 1/2$ there is no localization again. For $\nu < 1/2$ it is localized on the negative tension brane, as expected. In the case $\nu > 1/2$ we find that the normalization factor of the zero mode is $A \simeq \sqrt{k_1(\nu - 1/2)}$ which is the same as in the case of RS for $k = k_1$ (not surprisingly since it is strongly localized on the first brane).

The wavefunctions for the right-handed KK modes are given in terms of Bessel functions. For y lying in the regions $\mathbf{A} \equiv [0, L_1]$ and $\mathbf{B} \equiv [L_1, L_2]$, we have:

$$\hat{\Psi}^{(n)}\begin{Bmatrix} \mathbf{A} \\ \mathbf{B} \end{Bmatrix} = \begin{Bmatrix} \sqrt{\dfrac{g(z)}{k_1}}\left[A_1 J_{\nu+\frac{1}{2}}\left(\dfrac{m_n}{k_1}g(z)\right) + B_1 J_{-\nu-\frac{1}{2}}\left(\dfrac{m_n}{k_1}g(z)\right)\right] \\ \sqrt{\dfrac{g(z)}{k_2}}\left[A_2 J_{\nu+\frac{1}{2}}\left(\dfrac{m_n}{k_2}g(z)\right) + B_2 J_{-\nu-\frac{1}{2}}\left(\dfrac{m_n}{k_2}g(z)\right)\right] \end{Bmatrix} \tag{4.5}$$

with boundary conditions:

$$\begin{aligned} \hat{f}_n^{R\,\prime}(0^+) + \frac{k_1\nu}{g(0)}\hat{f}_n^R(0) &= 0 \\ \hat{f}_n^R({z_1}^+) - \hat{f}_n^R({z_1}^-) &= 0 \\ \hat{f}_n^{R\,\prime}({z_1}^+) - \hat{f}_n^{R\,\prime}({z_1}^-) - \frac{2\nu}{g(z_1)}\left(\frac{k_2 - k_1}{2}\right)\hat{f}_n^R(z_1) &= 0 \\ \hat{f}_n^{R\,\prime}({z_2}^-) + \frac{k_2\nu}{g(z_2)}\hat{f}_n^R(z_2) &= 0\,. \end{aligned} \tag{4.6}$$

The above boundary conditions result to a 4 x 4 homogeneous system for A_1, B_1, A_2 and B_2 which, in order to have a nontrivial solution, should have a vanishing determinant. This imposes a quantization condition from which we are able to extract the mass spectrum of the bulk spinor. The spectrum consists, apart from the chiral (right-handed) zero mode (massless) which was mentioned earlier, of a tower of Dirac KK modes.

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In this case in order to provide a solution to the hierarchy problem we have to arrange the distance between the first two branes so that we create the desired hierarchy w. In the present model we have an additional parameter which is the distance between the second and the third brane $x \equiv k_2(L_2 - L_1)$ For the region where $x \gtrsim 1$ we can find analytically that all the masses the KK tower (including the first's) scale the same way as we vary the length of the orbifold L_2 :

$$m_n = \zeta_n \; w k_2 \; e^{-k_2 L_2} \,, \tag{4.7}$$

where ζ_n is the n-th root of $J_{\nu-1/2}(x) = 0$.

In the region $x < 1$ the previous relation for the mass spectrum breaks down. This is expected since for $x = 0$ ($L_2 = L_1$) the "$++-$" model becomes "$+-$" (RS) and the quantization condition becomes approximately $J_{\nu-1/2}\left(\frac{m}{k_1} g(z_1)\right) = 0$, which is identical to the RS condition. So for $0 \leq x \leq 1$ the quantization condition (and thus the mass spectrum) interpolates between the previous two relations.

Let us now turn to the coupling between the SM neutrino which lives on the second positive brane with the bulk right-handed zero mode and the rest of the KK tower. We can easily derive that zero mode couples in that same way as in the RS case (the normalization of the zero mode is approximately the same):

$$\upsilon \; y_0 = \upsilon \; Y_5 \sqrt{k_1 \left(\nu - \frac{1}{2}\right)} \; \left(\frac{1}{(g(z_1))}\right)^{\nu-1/2} \simeq \upsilon \sqrt{\nu - \frac{1}{2}} \; w^{\nu-1/2} \,, \tag{4.8}$$

where, as we previously mentioned, $Y_5 \sim 1/\sqrt{k_1}$, $\upsilon \sim 10^2\,\mathrm{GeV}$, $\nu > 1/2$ and $w \equiv 1/g(z_1)$. On the other hand the coupling of the SM neutrino to bulk KK states is given by:

$$\upsilon \; y_n \sim \upsilon \; \sqrt{\nu} \left(\frac{k_2}{k_1}\right)^{3/2} \frac{8\zeta_n^2}{J_{\nu+1/2}\left(\zeta_n\right)} \; e^{-3x} \,, \tag{4.9}$$

where ζ_n is the n-th root of $J_{\nu-1/2}(x) = 0$.

All approximations become better away from $\nu = 1/2$, $x = 0$, and for higher KK levels. Note the strong suppression in the coupling scaling law. This rapid decrease, which is distinct among the models that we will consider, also appears in the coupling (to matter) behaviour of the graviton KK states and a detailed explanation can be found in ref. [33].

Thus from the above we conclude that for $\nu > 1/2$ the phenomenology of this model resembles, in the general characteristics, the one of RS. Of course in the present model there is an extra parameter, x, which controls the details of the masses and couplings of the KK states. Since the zero mode coupling is independent of x the general arguments of the previous section about creating small neutrino masses apply here as well, at least for small x . By increasing x we make the KK tower lighter, as we see from eq. (4.7), but we avoid large mixings between the SM neutrino and the left-handed bulk states due to the fact that the coupling between the SM neutrino and the right-handed bulk states drops much faster according with eq. (4.9).

Note that in the case $\nu < 1/2$ (negative values included) the zero mode will be localized on the negative brane and thus one could arrange the parameter x so that the exponential suppression of the bulk fermion's zero mode coupling on the second brane is such that gives small neutrino masses. Thus in this case it seems that we are able to solve the hierarchy problem by localizing the graviton on the first positive brane and in the same time create small neutrino masses by localizing the bulk fermion zero mode on the negative brane. Nevertheless, one should make sure that no large mixings are induced in this case.

5. Neutrinos in "$+-+$" model

We now turn to examine bulk spinors in the "$+-+$" model, which was analyzed in detail in ref. [31, 32]. The model consists of two positive tension branes placed at the orbifold fixed points and a third, negative brane which is freely moving in-between. SM field are considered to be confined on the second positive brane. Of course the presence of a moving negative brane is problematic since it gives rise to a radion field with negative kinetic term (ghost state) [35, 36] in the gravitational sector. Nevertheless we are interested in the general characteristics of this model . The interesting feature of this model is the bounce form of the warp factor which gives rise to an ultralight graviton KK state as described in ref. [31]. It was shown in ref. [34] that exactly this feature, of a bounce in the warp factor, can be reproduced even in the absence of negative branes in the "++" model where this is done by sacrificing the flatness of the branes (the spacetime on the branes in this case is AdS_4). Thus we will handle the "$+-+$" as a toy model since in this case there can be simple analytical calculations of the coupling etc. The general characteristics will persist in the "++" case.

We are interested to see if this configuration as well as an ultralight graviton supports an ultralight spinor field. In order to see this we should check the form of the potential of the Schrödinger equation that the right-handed component obeys. We can easily find that the potential is (for $z \geq 0$):

$$V_R(z) = \frac{\nu(\nu+1)k^2}{2[g(z)]^2} - \frac{\nu}{2g(z)} 2k\left[\delta(z) + \delta(z - z_2) - \delta(z - z_1)\right] \tag{5.1}$$

since $(\sigma'(y))^2 = k^2$ and $\sigma''(y) = 2kg(z)\left[\delta(z) + \delta(z - z_2) - \delta(z - z_1)\right]$. The convenient choice of variables in this case is:

$$z \equiv \begin{cases} \dfrac{2e^{kL_1} - e^{2kL_1 - ky} - 1}{k} & y \in [L_1, L_2] \\ \dfrac{e^{ky} - 1}{k} & y \in [0, L_1] \end{cases} \tag{5.2}$$

and the function $g(z)$ is defined as $g(z) \equiv k\left\{z_1 - ||z| - z_1|\right\} + 1$, where $z_1 = z(L_1)$.

As in the previous cases, the above potential always supports a (massless) zero mode with wavefunction of the form:

$$\hat{f}_0^R(z) = \frac{A}{[g(z)]^\nu}\,. \tag{5.3}$$

In this case the different localization behaviour as a function of ν is the following: For $\nu > 1/2$ the zero mode is localized on the positive branes (thus fails to be normalizable when we send the right positive brane to infinity but is normalizable when we send both negative and positive to infinity). For the case $\nu < 1/2$ the localization of the zero mode is on the negative tension brane, as expected. In the case $\nu > 1/2$ and for strong hierarchy w we find that the normalization factor of the zero mode is $A \simeq \sqrt{k_1(\nu - 1/2)}$ (Note that in the case of "weak" hierarchy one should be careful with the assumptions on which the approximations are based on e.g. for $w = 1$ the result must be divided $\sqrt{2}$). For the KK modes the solution is given in terms of Bessel functions. For y lying in the regions $\mathbf{A} \equiv [0, L_1]$ and $\mathbf{B} \equiv [L_1, L_2]$, we have:

$$\hat{f}_n^R \left\{ \begin{array}{c} \mathbf{A} \\ \mathbf{B} \end{array} \right\} = \sqrt{\frac{g(z)}{k}} \left[\left\{ \begin{array}{c} A_1 \\ B_1 \end{array} \right\} J_{1/2+\nu}\left(\frac{m_n}{k} g(z)\right) + \left\{ \begin{array}{c} A_2 \\ B_2 \end{array} \right\} J_{-1/2-\nu}\left(\frac{m_n}{k} g(z)\right) \right] \tag{5.4}$$

with boundary conditions:

$$\begin{aligned} \hat{f}_n^{R\,\prime}(0^+) + \frac{k\nu}{g(0)}\hat{f}_n^R(0) &= 0 \\ \hat{f}_n^R({z_1}^+) - \hat{f}_n^R({z_1}^-) &= 0 \\ \hat{f}_n^{R\,\prime}({z_1}^+) - \hat{f}_n^{R\,\prime}({z_1}^-) - \frac{2k\nu}{g(z_1)}\hat{f}_n^R(z_1) &= 0 \\ \hat{f}_n^{R\,\prime}({z_2}^-) - \frac{k\nu}{g(z_2)}\hat{f}_n^R(z_2) &= 0\,. \end{aligned} \tag{5.5}$$

The boundary conditions give a 4 x 4 linear homogeneous system for A_1, B_1, A_2 and B_2, which, in order to have a nontrivial solution should have vanishing determinant. This imposes a quantization condition from which we are able to extract the mass spectrum of the bulk spinor. The spectrum consists, apart from the chiral (right-handed) zero mode (massless) which was mentioned earlier, by a tower of Dirac KK modes. Nevertheless due to the fact that there are two positive tension branes present in the model there are now two "bound" states in a similar fashion with ref. [31, 34] (for $\nu > 1/2$). One is the the right-handed zero mode which is massless and it is localized on the positive brane placed at the origin of the orbifold and the second is the ultralight right-handed first KK state which is localized on the second positive brane placed at the other orbifold fixed point. This can be seen by examining the mass spectrum and the coupling behaviour of the first KK state in comparison with the rest of the tower.

Firstly let us examine the mass spectrum. In the case that we have a hierarchy w (where $w \equiv 1/g(z_2) = e^{-\sigma(L_2)}$) we can find appropriate analytical expressions for the mass spectrum.

For the first KK state

$$m_1 = \sqrt{4\nu^2 - 1}\; kw\; e^{-(\nu+1/2)x} \tag{5.6}$$

and for the rest of the tower

$$m_{n+1} = \xi_n\; kw\; e^{-x} \qquad n = 1, 2, 3, \dots , \tag{5.7}$$

where ξ_{2i+1} is the $(i+1)$-th root of $J_{\nu-1/2}(x)$ $(i = 0, 1, 2, \dots)$ and ξ_{2i} is the i-th root of $J_{\nu+1/2}(x)$ $(i = 1, 2, 3, \dots)$. The above approximations become better away from the $\nu = 1/2$, $x = 0$ and for higher KK levels n. The first mass is manifestly singled out from the rest of the KK tower as it has an extra exponential suppression that depends on the mass of the bulk fermion. By contrast the rest of the KK tower has only a very small dependence on the mass of the bulk fermion thought the root of the Bessel function $\xi_n = \xi_n(\nu)$ which turns out to be just a linear dependence in ν. Note there is a difference between the graviton ultralight state (discussed in [31, 32]) and this spinor state: In the case of gravity the unltralight KK state the mass scales as a function of x was e^{-2x}, on the other hand the scaling law in the case of the ultralight spinor is of the form $e^{-(\nu+1/2)x}$. From the above it seems that the latter can be done much lighter that the graviton first KK state for a given x by increasing the parameter ν. This is easy to understand since the role of the mass term, with the kink or multi-kink profile, is to localize the wavefunction $\hat{f}(z)$. By increasing the parameter ν all we do is to force the absolute value of the wavefunction of the first KK state and the massless right-handed zero mode to become increasingly similar to each other: For example, in the symmetric configuration, the difference between the zero mode and the first KK state wavefunctions comes from the central region of the "$+-+$" configuration, where the first KK state wavefunction is zero (since it is antisymmetric) thought the zero mode's is very small due to the exponential suppression of the wavefunction, but non zero. By increasing ν we force the value of the zero mode wavefunction at the middle point to get closer to zero and thus to resemble even more the first KK state, something that appears in the mass spectrum as the fact that the mass of the first KK state is approaching to zero. On the other hand the mass eigenvalues that correspond to the rest of the tower of KK states will increase linearly their mass by increasing the ν parameter since those are not bound states (the first mode has also such a linear dependence in ν but it is negligible compared with the exponential suppression associated with ν).

Now let us turn to the behaviour of the coupling of the zero mode and the KK states to matter living on the third (positive) brane. As in the previous cases the right-handed zero mode couples to SM left-handed neutrino as

$$\upsilon\ y_0 = \upsilon\ Y_5\ \sqrt{k\left(\nu - \frac{1}{2}\right)}\ \left(\frac{1}{g(z_2)}\right)^{\nu-1/2} \simeq \upsilon\ \sqrt{\nu - \frac{1}{2}}\ w^{\nu-1/2} \tag{5.8}$$

since $Y_5 \sim 1/\sqrt{k}$. From the above relationship we see that the coupling of the zero mode to SM neutrino will generally be suppressed by the hierarchy factor to some power, the power depending on the bulk fermion mass. This way one may readily obtain a very small coupling. The coupling of the zero mode is independent of x. This is another way to see the localization of this mode on the first brane (the normalization of the wavefunction is effectively independent of x). Since this model supports a second "bound state" (first KK state) which is localized on the second brane, we expect something similar to occur in the coupling behaviour of this state. Indeed, similarly to the graviton case [31, 32], we can show that the coupling of this state to the SM neutrino for fixed w is constant, i.e. independent of the x parameter. Taking in account the result of the graviton KK state $a_1 = 1/(wM_{Pl})$ and by comparing the graviton-matter and spinor matter coupling we can easily see that the coupling of this special mode will be of the order of the electroweak scale:

$$\upsilon\ y_1\ \simeq \sqrt{\nu - \frac{1}{2}}\ \upsilon\,. \tag{5.9}$$

Let us now consider the coupling of the rest right-handed KK states to the SM neutrino. We find that

$$\upsilon\ y_n\ \simeq \sqrt{\nu - \frac{1}{2}}\ \upsilon\ e^{-x} \tag{5.10}$$

for $n = 0, 1, 2\ldots$ From the above relationship we see that the rest of KK states will generally have exponentially suppressed coupling compared to the first special state.

The appearance of this special first ultralight and generally strongly coupled KK state, as in the graviton case, is going to have radical implication to the phenomenology of the model. Let us consider the following example suppose that $\nu = 3/2$ and that we also require a hierarchy of the order: $w \sim 10^{-15}$. In this case the zero mode's coupling is $\upsilon y_0 \sim \upsilon 10^{-15} \simeq 10^{-4}$ eV a result independent of the x parameter. On the other hand one can check that the rest of KK tower will have masses $m_n \simeq 10^3\ e^{-x}$ GeV (for $n = 2, 3\ldots$) with coupling $\upsilon y_n \sim \upsilon e^{-x}$. Up to this point the phenomenology associated with this model is similar to the RS case i.e. tiny coupling of the right-handed and generally heavy KK states with relatively strong coupling. However, taking in account the special KK state, we have the possibility of obtaining a much lighter state with large coupling (effectively independent of how light this state is). Having a light sterile state whose right-handed mode has strong coupling to the SM neutrino is potentially dangerous. In such a case we find that the dominant contribution to the mass eigenstate of the lightest mode (neutrino) $\nu_{\rm phys}$ will come from the left-handed component of this special sterile mode and not the weak eigenstate ν. Of course something like this is not acceptable since there are strict constrains for the mixing of SM neutrino to sterile states. Since the mass spectrum depends exponentially on the the x parameter which determines the distance between the branes, the above argument impose strong constraints on it's possible values.

Finally, in the case that $\nu < 1/2$ the bulk right-handed zero mode is localized on the negative tension brane. In this case a new possibility arises: By localizing the graviton wavefunction on the first brane we can explain the SM gauge hierarchy (by setting w to the desired value) and by localizing the bulk fermion zero mode on the negative tension brane to induce small neutrino mass (for appropriate value of the x parameter) for the SM neutrino which is confined on the right positive tension brane. Note that for $\nu < 1/2$ there is no special bulk spinor KK state and thus there is no immediate danger of inducing large neutrino mixing from such a state. However, the presence of the ultralight graviton KK state is restricting our parameter space as following: In order to solve the gauge hierarchy problem (assuming the SM on the third brane) we have to fix the one parameter of the model: $w \sim 10^{-15}$. Since the fermion zero mode is localized on the intermediate negative brane we have to arrange the distance between this and the third brane (for given ν), x, so that the coupling is sufficiently suppressed in order to give reasonable neutrino masses. This implies that $e^{-|\nu-1/2|x} \sim 10^{-13}$, if one considers the mass of the neutrino of the order of 10^{-1} eV. From the bounds derived in [32] we find, for $k \sim 10^{17}\,\text{GeV}$, that in order the ultralight graviton KK state not to induce modifications of gravity at distances where Cavendish experiments take place and not to give visible resonances to $e^+e^- \to \mu^+\mu^-$ processes, we should have $4.5 < x < 15$ or $x < 1$. The latter implies certain restrictions to the values of ν: $1.5 < -\nu < 6.2$ or $-\nu > 29.4$. In the above regions it is possible to simultaneously create the gauge hierarchy and small neutrino masses consistently, with the mechanism described earlier.

6. Neutrinos in "++" model

As we mentioned in the previous section the "++" model mimics the interesting characteristics of the "$+-+$" model without having any negative tension brane. Thus since the warp factor has a bounce form this model also supports an ultralight graviton as was shown in ref. [34], [37]–[39]. According to the previous discussion, we should expect that the model will support a ultralight sterile neutrino as well. This can be easily shown again by considering the form of the potential of the differential equation that the right-handed component is obeying, which will again turn out to be of the same form as the graviton.

Now it turns out that for the construction of such a "++" configuration, it is essential to have AdS_4 geometry on both branes (for details see ref. [34]). Thus in

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this case the background geometry is described by:

$$ds^2 = \frac{e^{-2\sigma(y)}}{(1 - H^2x^2/4)^2}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \tag{6.1}$$

where the corresponding inverse vielbein is given by

$$E^A_\alpha = \mathrm{diag}\bigg(e^{\sigma(y)}\left(1 - \frac{H^2x^2}{4}\right), e^{\sigma(y)}\left(1 - \frac{H^2x^2}{4}\right), e^{\sigma(y)}\left(1 - \frac{H^2x^2}{4}\right), \\ e^{\sigma(y)}\left(1 - \frac{H^2x^2}{4}\right), 1\bigg)\,. \tag{6.2}$$

Since now the brane is no longer flat the previous calculations for the action will be slightly modified. We briefly discuss these modifications. Following the same steps of the flat case, we write $\Psi = \Psi_R + \Psi_L$ where $\Psi_{R,L} = 1/2(1 \pm \gamma_5)\Psi$. Since the connection part of the lagrangian again doesn't give any contribution (since the vielbein is again symmetric) the action becomes:

$$S = \int \sqrt{\hat{G}}\; d^4x\; dy \bigg\{ e^{-3\sigma}\hat{E}^A_a\left(\bar{\Psi}_L i\gamma^a\partial_A\Psi_L + \bar{\Psi}_R i\gamma^a\partial_A\Psi_R\right) - e^{-4\sigma}m\frac{\sigma'(y)}{k}\times \tag{6.3}$$

$$\times\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L\right) - \frac{1}{2}\left[\bar{\Psi}_L(e^{-4\sigma}\partial_y + \partial_y e^{-4\sigma})\Psi_R - \bar{\Psi}_R(e^{-4\sigma}\partial_y + \partial_y e^{-4\sigma})\Psi_L\right]\bigg\},$$

where $\hat{E}^A_\alpha = (1 - \frac{H^2x^2}{4})\delta^A_\alpha$ with (a,A=0,1,2,3) is the induced vielbein and $\hat{G}$ the determinant of the induced metric. For convenience and in order to be able to use results of ref. [34], we set $A(y) \equiv e^{-\sigma(y)}$ where

$$A(y) = \frac{\cosh(k(y_0 - |y|))}{\cosh(ky_0)} \tag{6.4}$$

is the equivalent "warp" factor in this case, which is found by considering the configuration gravitationally. From the above relation it is clear that the "warp" factor has the desired bounce form, with a minimum at y_0. The position of the minimum, something that it is going to be important for the phenomenology of the model, is defined from the relationship $\tanh(ky_0) \equiv kV_1/|\Lambda|$, where V_1 is the tension of the first brane and Λ is the five dimensional cosmological constant. As we mentioned in the introduction, the profile of the mass term, $A'(y)/A(y)$, in the present model is not a simple combination of θ-functions. In particular there are two θ-function profiles near the orbifold fixed points which give rise to the positive branes, but there is also an intermediate kink profile of the form $-\tanh(k(y - y_0))$ which is associated with the presence of the bounce (this could give rise to a "$-$" brane as a limit , resulting to the familiar "$+-+$" configuration). Note that even though there is no brane at the position of the minimum of the "warp" factor the kink profile is expected to act in the same way, and thus induce localization of the fermion zero mode in specific regions of the parameter space, exactly as in the "$+-+$" model.

As in the flat-brane case, we can decompose the left-handed and right-handed fermion fields into KK states with nontrivial profile wavefunctions f_n^L, f_n^R (in respect to the fifth dimension) in order to be able to bring the lagrangian into the form

$$S = \sum_n \int d^4x \sqrt{-\hat{g}} \{ \hat{E}^A_\alpha \bar{\psi}_n(x) i\gamma^\alpha \partial_A \psi_n(x) - m_n \bar{\psi}_n(x)\psi_n(x) \} , \tag{6.5}$$

where the wavefunctions $f_n^L(y)$, $f_n^R(y)$ should obey the following equations

$$\begin{aligned} \left(-\partial_y + \frac{m}{k}\frac{A'(y)}{A(y)} \right) f_n^L(y) &= m_n A^{-1}(y) f_n^R(y) \\ \left(\partial_y + \frac{m}{k}\frac{A'(y)}{A(y)} \right) f_n^R(y) &= m_n A^{-1}(y) f_n^L(y) \end{aligned} \tag{6.6}$$

with the following orthogonality relations:

$$\int_{-L}^{L} dy A^{-1}(y) f_m^{L*}(y) f_n^L(y) = \int_{-L}^{L} dy A^{-1}(y) f_m^{R*}(y) f_n^R(y) = \delta_{mn} \,. \tag{6.7}$$

Again we solve the above system of differential equations by finding the second order differential equation that it implies for the right-handed component of the spinor. It is always possible to make the coordinate transformation from y coordinates to z coordinates related through: $dz/dy = A^{-1}(y)$ and bring the differential equations in the familiar form:

$$\left\{ -\frac{1}{2}\partial_z{}^2 + V_R(z) \right\} \hat{f}_n^R(z) = \frac{m_n^2}{2} \hat{f}_n^R(z) \,, \tag{6.8}$$

where we have defined $\hat{f}_n^R(z) = f_n^R(y)$ with

$$\begin{aligned} V_R(z) &= \frac{\nu}{2} A(y) A''(y) + \frac{\nu^2}{2} (A'(y))^2 \\ &= -\frac{\nu^2 \tilde{k}^2}{2} + \frac{\nu(\nu+1)\tilde{k}^2}{2} \frac{1}{\cos^2\left(\tilde{k}(|z| - z_0)\right)} - \\ &\quad -k\nu \left[\tanh(ky_0)\delta(z) + \frac{\sinh(k(L-y_0))\cosh(k(L-y_0))}{\cosh^2(ky_0)} \delta(z - z_1) \right] \end{aligned} \tag{6.9}$$

with $\tilde{k}$ defined as $\tilde{k} \equiv k/\cosh(ky_0)$. The new variable z is related to the old one y through the relationship:

$$z \equiv \mathrm{sgn}(y) \frac{2}{\tilde{k}} \left[\arctan\left(\tanh\left(\frac{k(|y| - y_0)}{2} \right) \right) + \arctan\left(\tanh\left(\frac{ky_0}{2} \right) \right) \right] . \tag{6.10}$$

Thus in terms of the new coordinates, the branes are placed at $z_1 = 0$ and z_L, with the minimum of the potential at $z_0 = (2/\tilde{k}) \arctan(\tanh(ky_0/2))$. Also note that with this transformation the point $y = \infty$ is mapped to the finite point $z_\infty = (2/\tilde{k}) [\pi/4 + \arctan(\tanh(ky_0/2))]$.

We can now proceed to the solution of the above equations for the right-handed components, while the left-handed wavefunctions can be easily evaluated using eq. (2.10) (taking in account the definition $A \equiv e^{-\sigma(y)}$ and the change of variables). The zero mode wavefunction is given by:

$$\hat{f}_0^R(z) = \frac{C}{[\cos(\tilde{k}(z_0 - |z|))]^\nu}\,, \tag{6.11}$$

where C is the normalization factor. If we send one of the two branes to infinity (i.e. $z_1 \to z_\infty$) and at the same time keep z_0 fixed we find that the zero mode is normalizable only in the cases where $\nu < 1/2$. In the other cases ($\nu \geq 1/2$) the wavefunction fails to be normalizable due to the fact that it is too singular at z_∞. Note though that in the case where $\nu < 1/2$ the first KK will not be special (i.e. will have almost the same behaviour as the rest of the KK tower). We also note that again the zero mode is chiral i.e. there is no solution for f_n^L when $m_0 = 0$ that can satisfy the boundary conditions (antisymmetric wavefunction). From the above we see that the localization behaviour of zero mode in the "++" model is the same as in the "+ − +" model. Note that for $\nu < 1/2$ the zero mode will be localized near y_0 despite the absence of any brane at that point. This is because, as we mentioned, the $-A'(y)/A(y)$ factor, which can be considered as the vacuum expectation value of a scalar field, has a kink profile in the neighbourhood of y_0 which induces the localization (this kink becomes the negative tension brane in the flat brane limit). By considering cases with $m_n \neq 0$, we find the wavefunctions for the KK tower :

$$\hat{f}_n^R(z) = \cos^{\nu+1}(\tilde{k}(|z|-z_0))\Bigg[C_1\ F\left(\tilde{a}_n, \tilde{b}_n, \frac{1}{2}; \sin^2(\tilde{k}(|z|-z_0))\right) + C_2\ |\sin(\tilde{k}(|z|-z_0))| \times \\ \times F\left(\tilde{a}_n + \frac{1}{2}, \tilde{b}_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}(|z|-z_0))\right)\Bigg]\,, \tag{6.12}$$

where

$$\begin{aligned} \tilde{a}_n &= \frac{\nu+1}{2} + \frac{1}{2}\sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \nu^2} \\ \tilde{b}_n &= \frac{\nu+1}{2} - \frac{1}{2}\sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \nu^2}\,. \end{aligned} \tag{6.13}$$

The boundary conditions are given by:

$$\begin{aligned} \hat{f}_n^{R\,\prime}(0^+) + k\nu \tanh(ky_0)\hat{f}_n^R(0) &= 0 \\ \hat{f}_n^{R\,\prime}(z_L{}^-) - k\nu\frac{\sinh(k(L-y_0))}{\cosh(ky_0)}\hat{f}_n^R(z_L) &= 0 \end{aligned} \tag{6.14}$$

the above conditions determine the mass spectrum of the KK states. By studying the mass spectrum of the KK states it turns out that it has a special first mode similar to the one of the "$+-+$" model as expected. For example, for the symmetric configuration ($w=1$), by approximation we can analytically find the following expressions for the mass of this special state :

$$m_1 = 2\sqrt{4\nu^2+3}\;k\;\left(e^{-ky_0}\right)^{\nu+1/2}\;. \tag{6.15}$$

In contrast, the masses of the next levels are given by the formulae: for odd states

$$m_n = 2\sqrt{(n+1)(n+1+\nu)}\;k\;e^{-ky_0} \tag{6.16}$$

with $n=0,1,2,\ldots$, and for even states

$$m_n = 2\sqrt{\left(n+\frac{3}{2}\right)\left(n+\frac{3}{2}+\nu\right)}\;k\;e^{-ky_0} \tag{6.17}$$

with $n=0,1,2,\ldots$

Again the mass of the first KK state is manifestly singled out from the rest of the KK tower as it has an extra exponential suppression that depends on the mass of the bulk fermion. The above characteristics persist in the more physically interesting asymmetric case ($w \ll 1$). In this case we find $m_1 \sim m_1^0(\nu)\;kw\;(e^{-kx})^{\nu+1/2}$ and $m_n \sim m_n^0(\nu)\;kw\;e^{-kx}$ where $x = L_1 - y_0$ is the distance of the second brane from the minimum of the warp factor, and where $m_1^0 \sim 1$, m_n^0 has a linear dependence on n (for higher levels) and a $\sim\sqrt{\nu}$ dependence . The first KK state is localized on the second positive brane and, as in the case of "$+-+$" model, its coupling to SM neutrinos remains constant if we keep the hierarchy parameter w fixed. The phenomenology of this model will be similar to the "$+-+$" model and thus we do not consider it separately.

7. Bigravity and bulk spinors

Bigravity [31]–[34] (multigravity [40]–[33]) is the possibility that gravitational interactions do not exclusively come from a massless graviton , but instead they can be the net effect of a massless graviton and one or more KK states (continuum of KK states) or even a single massive KK mode [34, 37] without conflict with General Relativity predictions [42, 43]. This is based on the different scaling laws of the mass between the first and the rest of KK states. Since the first graviton KK state has mass with an additional exponential suppression we can realize the scenario that the first KK state is so light that its wavelength is of the order of the observable universe and thus any observable effect of its non-vanishing mass to be out of the experimental reach, and in the same time the rest of the KK tower has masses above the scale that Cavendish experiments have tested newtonian gravity at small distances.

In the two previous sections we have shown that in the case of models, where Bigravity can be realized, there is also an ultralight KK that corresponds to the bulk

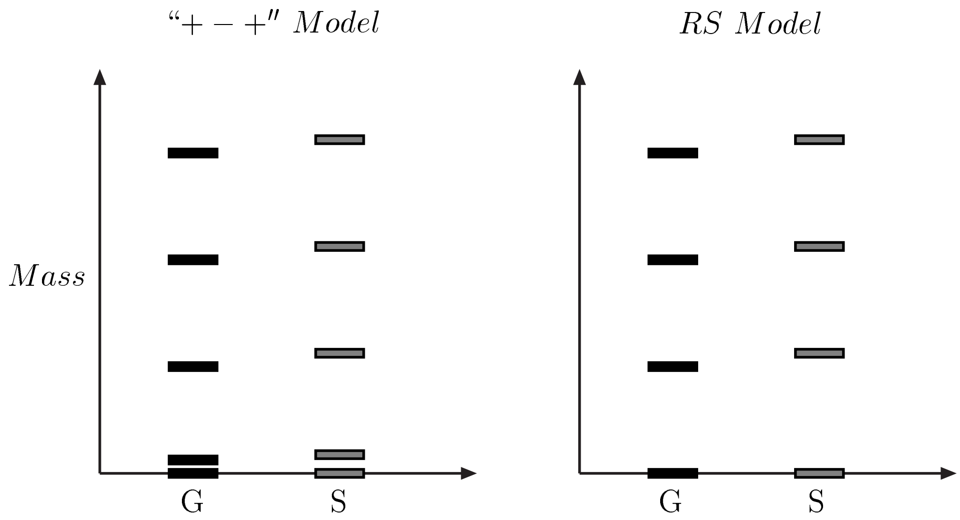

Figure 2: On the left, the mass spectrum of the graviton (G) (first column) and bulk spinor (S) (second column) KK states in the “+ − +”-Bigravity model. On the right, for comparison, the corresponding spectrum for the case of RS model. The figures are not in scale and the details of the bulk spinor mass spectrum depend on the additional parameter ν and thus in the above figure we have assumed $\nu \simeq 3/2$ (for higher value of ν we could have the fermion first KK state to be lighter that the first graviton KK state).

spinor assuming the existence of the mass term for the bulk spinor that appears in eq. (2.2) (with $\nu > 1/2$). The subject of this section is to investigate if the two above possibilities are compatible: Can we have a Bigravity scenario and a consistent neutrino phenomenology? For the sake of simplicity the discussion below will be concentrated to the “+ − +” model but, analogous arguments should apply to the case of “++” model.

Let us review briefly the Bigravity scenario. The graviton ultralight first KK state has mass $m_1^{(G)} = 2wk\ e^{-2x}$ and coupling $a_1^{(G)} = 1/(wM_{Pl})$. The rest of the KK tower has masses :

$$m_{n+1}^{(G)} = \xi_n\ kw\ e^{-x} \qquad n = 1, 2, 3, \dots , \tag{7.1}$$

where ξ_{2i+1} is the $(i+1)$-th root of $J_1(x)$ $(i = 0, 1, 2, \dots)$ and ξ_{2i} is the i-th root of $J_2(x)$ $(i = 1, 2, 3, \dots)$. The couplings of these states scale as $a_n \propto e^{-x}$. In order to achieve the Bigravity scenario, as we mentioned in the beginning of this section, we have the following constrains on the range of masses of the KK states: $m_1^{(G)} < 10^{-31}$eV or $m_2^{(G)} > 10^{-4}$eV (where Planck suppression is considered for the “continuum ” of states above 10^{-4}eV). Our exotic scheme corresponds to the choice $m_1^{(G)} \approx 10^{-31}$eV and $m_2^{(G)} > 10^{-4}$eV. In this case, for length scales less than 10^{26}cm gravity is generated by the exchange of *both* the massless graviton and the first KK mode. This implies, (taking into account the different coupling suppressions of the massless graviton and the first KK state) that the gravitational coupling as we measure it is related to the parameters of our model by:

$$\frac{1}{M_{\rm Pl}^2}=\frac{1}{M_5^2}\left(1+\frac{1}{w^2}\right)\approx\frac{1}{(wM_5)^2}\Rightarrow M_{\rm Pl}\approx wM_5\,. \tag{7.2}$$

We see that the mass scale on our brane, wM_5, is now the Planck scale so, although the "warp" factor, w, may still be small (i.e. the fundamental scale $M_5 \gg M_{Planck}$), we do not solve the Planck hierarchy problem. Using the equations for the mass spectrum and assuming as before that $k \approx M_5$, we find that $m_1 = 2\ kw\ e^{-2x} \approx M_{Planck}e^{-2x}$. For $m_1 = 10^{-31}$eV we have $m_2 \approx 10^{-2}$eV. This comfortably satisfies the bound $m > 10^{-4}$eV.

Now let us see what this implies for the neutrino physics. Let us first consider the case where $\nu > 1/2$. In this case, as mentioned above, there will be also an ultralight bulk fermion KK mode. By forcing the graviton first mode to have a tiny mass we also force the first spinor KK state to become very light (even in the best case where $\nu \to 1/2$ this state will have mass of the order of a fraction of eV. For larger values of ν the state becomes even lighter) since they are related through: $m_1 \sim e^{(3/2-\nu)x}\ m_1^G$. This is unacceptable since this mode has a constant coupling of the order of the weak scale, v, which will induce large mixing of the neutrino with the left-handed component of this state. However note that in the limit $\nu \to 1/2$ the special first fermion KK mode will become a normal one losing it's localization and thus the above may not apply. Unfortunately, this is not the case as in this limit the fermion zero mode is delocalized (for $\nu = 1/2$ the coupling is constant across the extra dimension) giving no possibility of inducing small neutrino masses (since the compactification volume is very small).[12]

Despite the severe constraints in the above scenario, due to the presence of the ultralight bulk fermion KK state, by no means the Bigravity scenario is excluded in the case of $\nu > 1/2$ since one can always consider the possibility of placing the SM on a brane (with tiny tension so that the background is not altered) between the negative and the second positive brane, so that the coupling of the bulk fermion first KK state to SM neutrinos is sufficiently small while a part of gravitational interactions will still be generated from the ultralight graviton first KK state.

Let us turn now to the case $0 \leq \nu < 1/2$. In this case there is no special bulk spinor KK state and thus the above arguments do not apply. In this case the bulk fermion zero mode is localized on the negative tension brane. Nevertheless, in order for the Bigravity scenario to be possible we should have $x \simeq 60$. If we now try to generate neutrino masses with the mechanism described in the first section we will

[12] Trying to use this window of the parameter space seems like a fine-tuning though since one needs to delocalize the fermions first KK state enough in order to have small coupling to SM neutrino and on the other hand to prevent the delocalization of the fermion zero mode with the same mechanism ($\nu \to 1/2$).

find that the coupling of the zero mode to SM neutrino will be by far too small to provide consistent results. Thus in this case although Bigravity is realized we cannot generate neutrino masses consistently.

The case of $\nu = 1/2$ is of no interest since in this case there is no localization (the fermion zero mode has constant coupling across the extra dimension). The case where $\nu < 0$ resembles the $0 \leq \nu < 1/2$ case, with the only difference that the localization on negative branes will be even sharpner, making the situation even worse.

8. Discussion and conclusions

We have studied bulk fermion fields in various multi-brane models with localized gravity. The chiral zero mode that these models support can be identified as a right-handed sterile neutrino. In this case small neutrino Dirac masses can naturally appear due to an analogous (to graviton) localization of the bulk fermion zero mode wavefunction without invoking a see-saw mechanism. For models in which the localization of the fermion zero mode is induced by the same scalar field that forms the branes the localization behaviour of this mode can resemble the graviton's at least in a region of parameter space. The latter implies that the "$+-+$", "$++$" models can support, in addition to the ultralight graviton KK state, an ultralight localized and strongly coupled bulk fermion KK mode. This fermion state, when exists, imposes even more severe constrains on the parameter space of "$+-+$" and "$++$" models. In the case that one requires the Bigravity be realized the light fermion KK mode can induce too large mixing between the neutrino and the KK tower and thus it restricts even more the allowed parameter space of the relevant models.

As a general remark we see that the appearance of Multi-localization [44] in the Multi-Brane world picture and its relation to the existence of ultralight states in the KK spectrum is not a characteristic of the graviton only, but can also occur in spin 0, 1/2 and 3/2 bulk states with appropriate bulk mass terms [45]. Vector fields on the other hand can have also ultralight states but in this case a coupling to a bulk scalar field is necessary in order to achieve localization [25].

The appearance of ultralight KK states with the additional characteristic, in the case of spin 1/2 fields, where the left-handed and the right-handed components are localized in different places in the extra dimension, can give new interesting possibilities which will be discussed in another publication [45].

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